ACTIVE OPTICAL SYSTEMS

AN023

Comparison of Strehl Ratio, Marechal Approximation, and Power in a Bucket Calculations

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Abstract

This application note explores the relationship between Strehl Ratio (SR), the Marechal approximation, and power in a bucket (PIB) metrics for a circular beam with select Zernike and sinusoidal aberrations.

Model Setup

This model was run in Matlab using some existing functions for things like Zernike polynomials, far-field propagation calculations, creating grids, and plotting. The appendix shows the code for the model with sinusoidal gratings, but does not include the support scripts, although those should be easily duplicated. We used two separate scripts for analysis of Zernikes and sinusoidal phase gratings.

In each of the scripts, the outer most loop scans over different aberration amplitudes. Each aberration was scaled to make the root-mean-square (RMS) amplitude the same for different aberration shapes. The next inner loop scaled over aberration shape. In each shape-amplitude combination, the aberration is applied to a circular beam on a mesh with a factor of two guard-band (dxy = 2 * Dap / nxy). The Strehl ratio is calculated using the integral form of the equation. The far-field intensity of the aberrated beam is determined and then the power in the bucket calculation is performed. The power in the bucket to total power ratio is then stored for a 1x, 1.5x and 2x diffraction limited bucket size. The Marechal approximation was calculated using,

$$Strehl_{Marechal} = \exp\left(-\phi_{RMS}{}^{2}\right)$$

where ϕ_{RMS} is the RMS aberration amplitude.

In each of these studies, the wavelength was 1 µm, the circular top-hat beam was 1 cm in diameter, the mesh was sampled with 256 points, and the beam was propagated 25 m to the focus.

Results with Zernike Polynomial Aberrations

We studied Zernikes from the second order (#3) to the 33rd Zernike (n-l ordering) and then also Zernike #100. Figure 1 shows the results for the focus aberration. Figure 2 shows the results for the 100th Zernike, which is a 5th-radial-order polynomial with 10 circular segments. In general, for all of the Zernikes, we found Strehl ratio and Marechal approximation matched well up to about 0.15 waves RMS amplitude. The power in the diffraction limited bucket matched reasonably well with the Strehl ratio and Marechal approximation up to this point as well. In general, the agreement between all of these metrics increased as the aberration spatial frequency increased, as is evidenced by the good agreement in Figure 2.

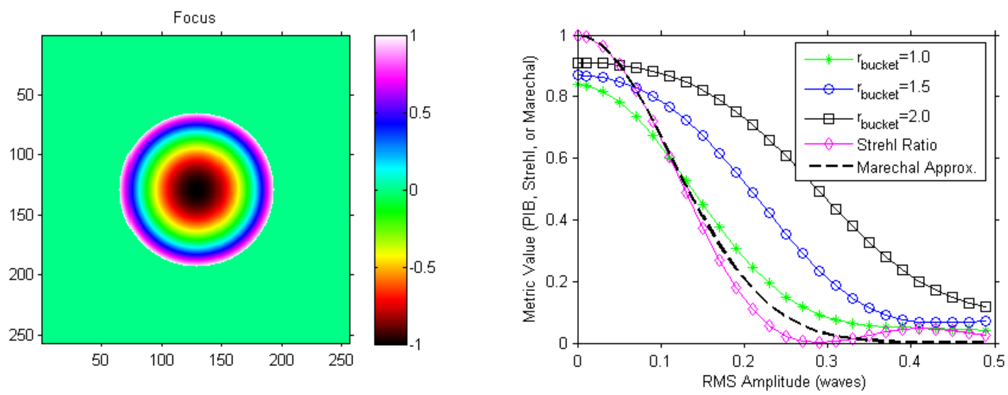

Figure 1 - Results for a Focus Aberration

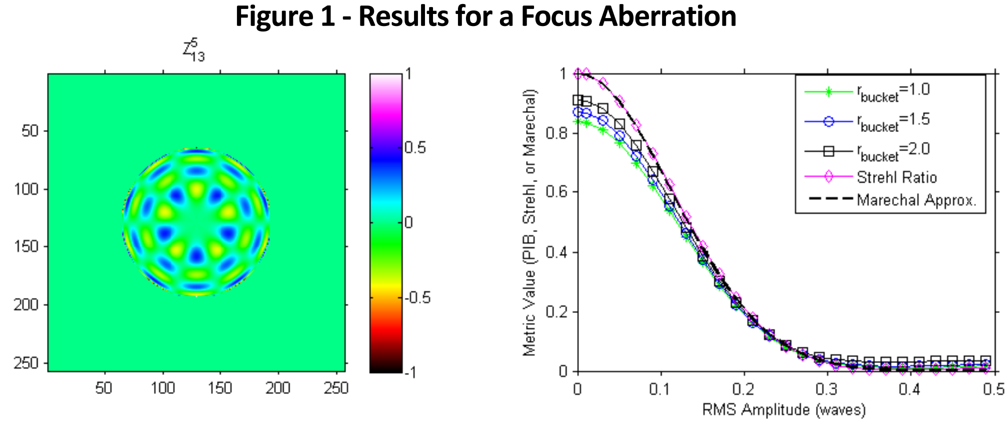

Figure 2 - Results for the 100th Zernike

Results with Sinusoidal Phase Grating Aberrations

In the case of the sinusoidal gratings, we studied gratings with spatial periods equal to the aperture diameter divided by a factor between 1 and 10 in unit increments. Figure 3 and Figure 4 show the results for the lowest and highest spatial frequencies we studied respectively. The results, in general, were consistent with the conclusions of the Zernike polynomial study. The low spatial frequency aberrations were the most different than the Strehl ratio and Marechal approximation results, but the power in the diffraction limited bucket curve seemed to match reasonably well with the Strehl results.

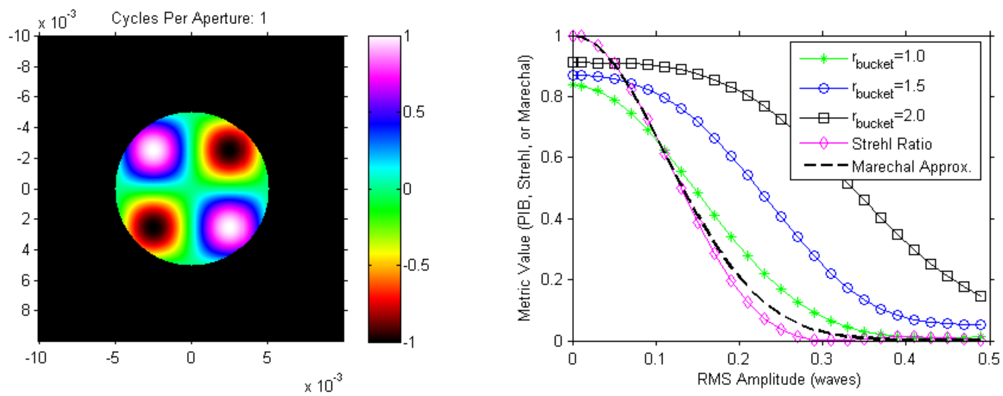

Figure 3 - Results with the lowest spatial frequency grating

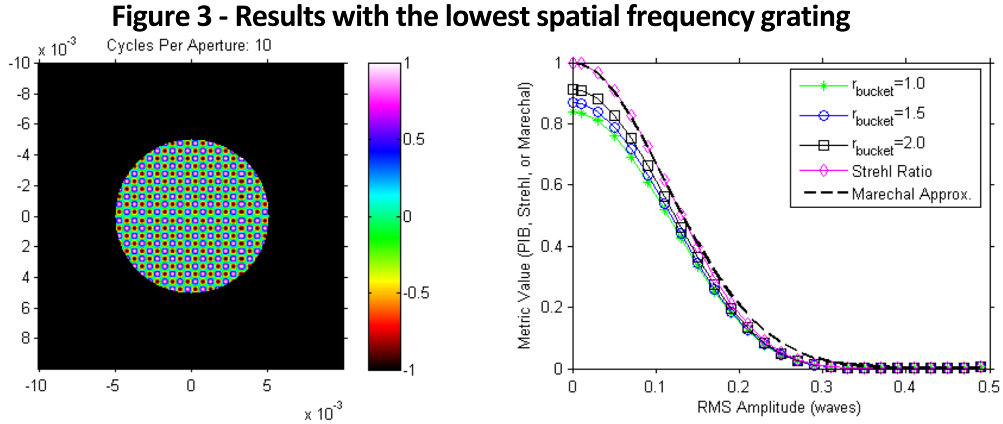

Figure 4 - Results with the highest spatial frequency grating

For the sinusoidal aberrations, we also looked at the difference between the PIB and the Strehl ratio calculations. Figure 5 shows the result for the highest spatial frequency. There is an initial difference due to the light diffracted beyond the bucket even with no aberration amplitude, but this only decreases as the aberration amplitude increases. The maximum value of this is about 16% for the 1x diffraction limited bucket size.

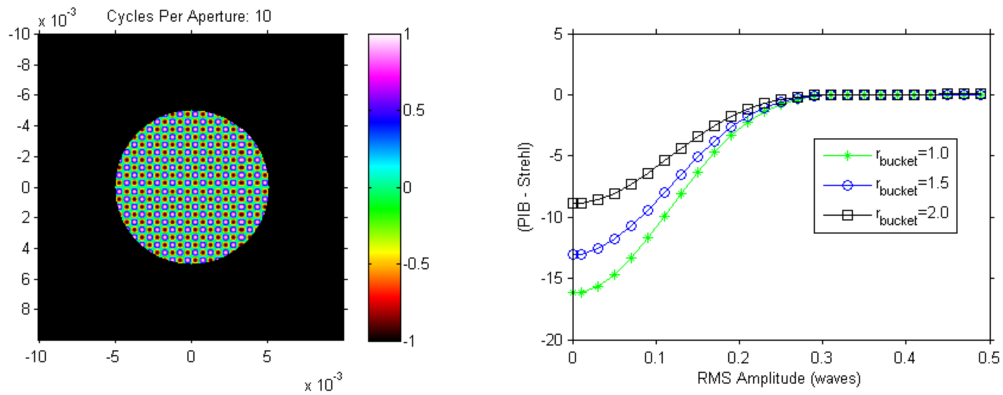

Figure 5 - Difference between PIB and Strehl Ratio for the highest spatial frequency aberration

Conclusions

In this application note, we have shown the great deal of similarity between the Strehl ratio, the Marechal approximation, and the power in a diffraction limited bucket. We intend to continue this line of study and add to this application note as time permits.

Appendix: Comparison Code for Sinusoids

```
setup;
dbg=0; ppt=1;

nxy = 512;
Dap = 1e-2;
wavelength = 1.0e-6;
dxy = Dap*2/nxy;
rzern = Dap/2;
SF = 5.0;   %far field scaling

%scanning parameters
rBucket = [1.0 1.5 2];
zVec = ([1:1:10]);
ampSFv = [0 0.01:0.02:0.5];

if (ppt)
    TitleSlidePowerPoint('Study of comparison of Strehl, Marechal, and Power
in a Bucket');
      xxx=whos;
      for ii=1:length(xxx);
          eval(sprintf('val=%s;',xxx(ii).name));
          txt{ii} = sprintf('%s = %f',xxx(ii).name,val);
      end;
      TextToPowerPoint(txt,'Variables',12,16);
      clear xxx;
      clear txt;
      clear val;

      TextFileToPPT('MarechalStrehlPIBStudy_Sinusoids.m');
end;

%calcualte ideals
g=makeGrid(nxy,dxy);
E0 = aperture(nxy,dxy,Dap/2);
[Effi,dxout,xout,zff]=FarField(E0,dxy,wavelength,SF);
rdl = 1.22 * zff * wavelength / Dap;
[rv,Ps,Pseg]=PowerInBucket(abs(Effi).^2,dxout);
PbIdeal = Ps ./ max(Ps);

%RMS aperture
ap = aperture(nxy,dxy,rzern,0,0,0,1);

b=0;
for ampSF=ampSFv;
    nf([375    71    966    617]);
    b=b+1;
    rmsamp = wavelength * ampSF;
    c = 0;
    for ii=zVec;
        c=c+1;
```

```
        Kx = 2.0*pi/(Dap/ii);
        z = sin(g.xx.* Kx) .* sin(g.yy.*Kx);
        %nf; show(z); return;

        %scale RMS
        z = z .* rmsamp./(NonZeroRMS(z.*ap));
        E = E0 .* exp(1j.*(2.0*pi/wavelength).*z);

        SR(b,c) = Strehl(E);

        [Eff,dxout,xout,zff]=FarField(E,dxy,wavelength,SF);

        rdl = 1.22 * zff * wavelength / Dap;
        %[P,Ps,x]=PIB(Eff,dxy);
        [rv,Ps]=PowerInBucket(abs(Eff).^2,dxout);
        Pb = Ps ./ max(Ps);
        rb = rv./rdl;
        index(c)=ii;
        for jj=1:length(rBucket);
            [mv,mi(jj)]=min(abs(rb - rBucket(jj)));
            Pbs(jj,b,c) = Pb(mi(jj));
        end;

        if (dbg)
            clf;
            subplot(2,2,1); show(abs(E)); drawnow; title(ii);
            subplot(2,2,2); show(z); drawnow; title('Phase');
            subplot(2,2,3); show(abs(Eff)); drawnow; title('Far-
Field');
            subplot(2,2,4); plot(rb,Pb);
            hold on;
            plot(rb,PbIdeal,'k--');
            ax=axis; ax(2)=5; axis(ax);
            drawnow; title('PIB');
            pause(0.001);
        end;
    end;
    close gcf;

    nf;
    clear leg;
    for jj=1:size(Pb,1);
        Pbv = Pbs(1,b,:);
        Pbv = Pbv(:);
        plot(index,Pbv,getLineSpec(jj));
        hold on;
        leg{jj}=sprintf('r=%.1f',rBucket(jj));
    end;
    plot(index,SR(b,:),'g^-');
    leg{end+1}=sprintf('Strehl');
    plot(index,index.*0 + exp(-1* ((ampSF*2*pi).^2)))
    leg{end+1}=sprintf('Marechal');
    ax=axis; ax(3)=0; axis(ax);
    legend(leg);
    xlabel('Zernike');
    ylabel('Value');
```

```
end;

%% display final results
nf;
plot(ampSFv,exp(-1*((ampSFv*2*pi).^2)),'k--*','LineWidth',2);
hold on;
for ii=1:3; %size(Pbs,3);
    Pbv = Pbs(1,:,ii);
    Pbv = Pbv(:);
    plot(ampSFv,Pbv,getLineSpec(ii));
    plot(ampSFv,SR(:,ii),getLineSpec(ii,'ColorAndMarker'));
end;
plot(ampSFv,exp(-1*((ampSFv*2*pi).^2)),'k--*','LineWidth',2);
xlabel('RMS Amplitude (waves)');
ylabel('Ampliutde');
legend('Marechal','P Ratio','Strehl');

%% results by aberration shape
nf([291         363        1050         325]);
for ii=1:length(zVec);
    clf;
    subplot(1,2,1);
    Kx = 2.0*pi/(Dap/zVec(ii));
    z = sin(g.xx.* Kx) .* sin(g.yy.*Kx);
    show(z.*ap);
    title(sprintf('Cycles Per Aperture: %i',zVec(ii)));

    subplot(1,2,2);
    clear leg;
    for jj=1:length(rBucket);
        Pbv = Pbs(jj,:,ii);
        Pbv = Pbv(:);
        plot(ampSFv,Pbv,getLineSpec(jj));
        hold on;
        leg{jj} = sprintf('r_b_u_c_k_e_t=%.1f',rBucket(jj));
    end;
    plot(ampSFv,SR(:,ii),getLineSpec(length(rBucket)+1));
    leg{end+1}=sprintf('Strehl Ratio');
    plot(ampSFv,exp(-1*((ampSFv*2*pi).^2)),'k--','LineWidth',2);
    leg{end+1}=sprintf('Marechal Approx.');
    legend(leg,'Location','Best');
    drawnow;
    if (ppt)
        ToPPT();
    end;
end;
```